WAYS OF GENERALIZATION : WAYS TO DEVELOP MATHEMATICAL PRACTICES IN ELEMENTRY SCHOOL

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Research Questions:

- To what extent do the curriculum tasks and accompanying teacher material included in specific lessons in the teacher's edition for the third grade textbooks create the potential for teachers to mediate students engagement in describing, extending, and making generalizations about geometric and numeric patterns?
- How is the generalization process represented in the curriculum tasks and accompanying teacher materials across multiple lessons on multi-digit addition and subtraction in the teacher's edition for the third grade textbooks?

Table 1. Core aspects and Strands of Generalization

Strands / Core aspects	Generalization of arithmetic and quantitative reasoning (1)	Functions Relations Variations (2)	Modeling (3)
A Successive processes of record making and action on those records	A1	A2	A3
B Lifting up of repeated actions on symbols into syntax	B1	B2	B3

Table 2. The coding system

Strands / Core aspects		Generalization of arithmetic and quantitative reasoning (1)				Functions Relations Variations (2)				Modeling (3)			
	Facets	N	V	K	C	N	V	K	C	N	V	K	C
A Successive processes of record making and action on those records	*Geometric*	A1 Ge N	A1 Ge V	A1 Ge K	A1 Ge C	A2 Ge N	A2 Ge V	A2 Ge K	A2 Ge C	A3 Ge N	A3 Ge V	A3 Ge K	A3 Ge C
	Numeric	A1 Nu N	A1 Nu V	A1 Nu K	A1 Nu C	A2 Nu N	A2 Nu V	A2 Nu K	A2 Nu C	A3 Nu N	A3 Nu V	A3 Nu K	A3 Nu C
	Other	A1 Vi N	A1 Vi V	A1 Vi K	A1 Vi C	A2 Vi N	A2 Vi V	A2 Vi K	A2 Vi C	A3 Vi N	A3 Vi V	A3 Vi K	A3 Vi C
B Lifting up of repeated actions on symbols into syntax	*Geometric*	B1 Ge N	B1 Ge V	B1 Ge K	B1 Ge C	B2 Ge N	B2 Ge V	B2 Ge K	B2 Ge C	B3 Ge N	B3 Ge V	B3 Ge K	B3 Ge C
	Numeric	B1 Nu N	B1 Nu V	B1 Nu K	B1 Nu C	B2 Nu N	B2 Nu V	B2 Nu K	B2 Nu C	B3 Nu N	B3 Nu V	B3 Nu K	B3 Nu C
	Other	B1 Vi N	B1 Vi V	B1 Vi K	B1 Vi C	B2 Vi N	B2 Vi V	B2 Vi K	B2 Vi C	B3 Vi N	B3 Vi V	B3 Vi K	B3 Vi C

Legend: N=conventional notation; V=verbal; K=kinesthetic; C=colloquial

DISCUSSION

1 Numbering Repeating Patterns

15 MIN CLASS

Math Focus Points for Discussion

- Identifying the unit of a repeating pattern
- Associating counting numbers with elements of a pattern

In this discussion, students review ideas from Grade 2 about how repeating patterns are constructed by iterating a unit (the part of the pattern that repeats). They are introduced to the idea of applying numbers to a repeating pattern of colored cubes.[1]

Introduce this Investigation briefly.

Some of you may remember when you worked with color patterns, using the connecting cubes in Grade 2. We're going to work for a few days on repeating patterns. By numbering the cubes, we'll be able to use the numbers to describe how a repeating pattern changes. We'll also figure out what color certain cubes will be—such as the 50th cube or the 100th cube—without actually building the pattern all the way to 50 or 100 cubes.

Show students the 12-cube train you prepared with the pattern: red-blue-green, red-blue-green, red-blue-green, red-blue-green.

This is a repeating pattern. What do you see in this pattern? How would you describe it?

A1ViV

Use students' language and observations to bring out the idea that the unit for this pattern is three cubes long and is *red-blue-green.* If students worked with the Grade 2 Patterns and Functions unit last year, you can refer to that experience.

Last year when you worked on patterns, you talked about the unit of a repeating pattern. What is the unit of this pattern? How many cubes are there in the part that repeats over and over?

B1ViV

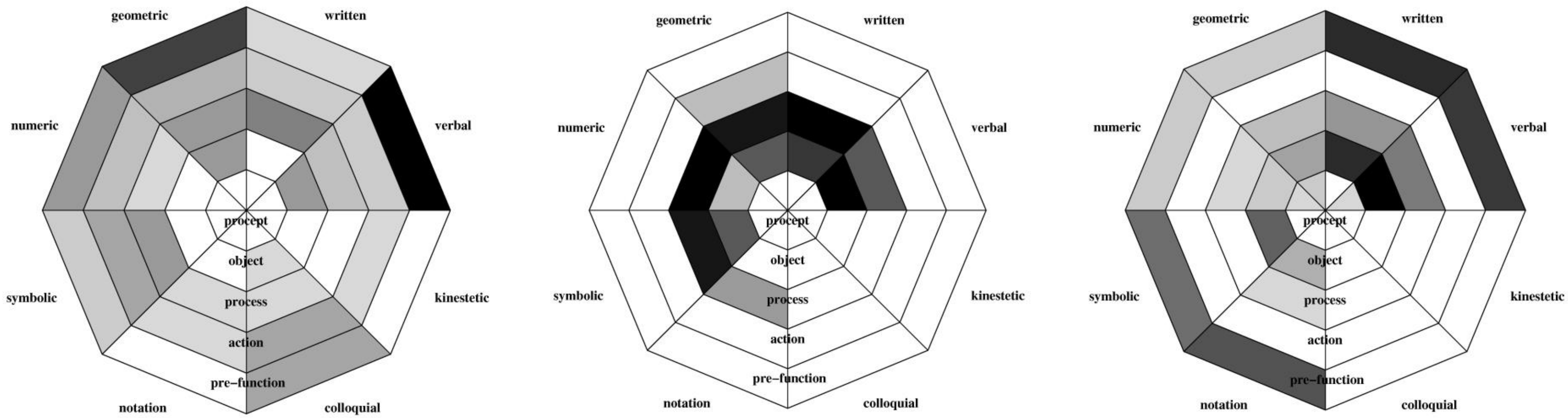
Ask students whether they can come up with other patterns, using numbers or letters or body movements that have the same kind of pattern as this color pattern. For example, sometimes students suggest "1-2-3, 1-2-3" or "a-b-c, a-b-c" or come up with a body movement pattern such as slap knees-clap-tap shoulders. It is helpful to show how the red-blue-green pattern matches these other patterns by having the class chant "red,

A1NuV
A1ViK
A1ViC

(continued on next page)

Professional Development

[1] **Teacher Note:** Repeating Patterns and Counting Numbers, p. 124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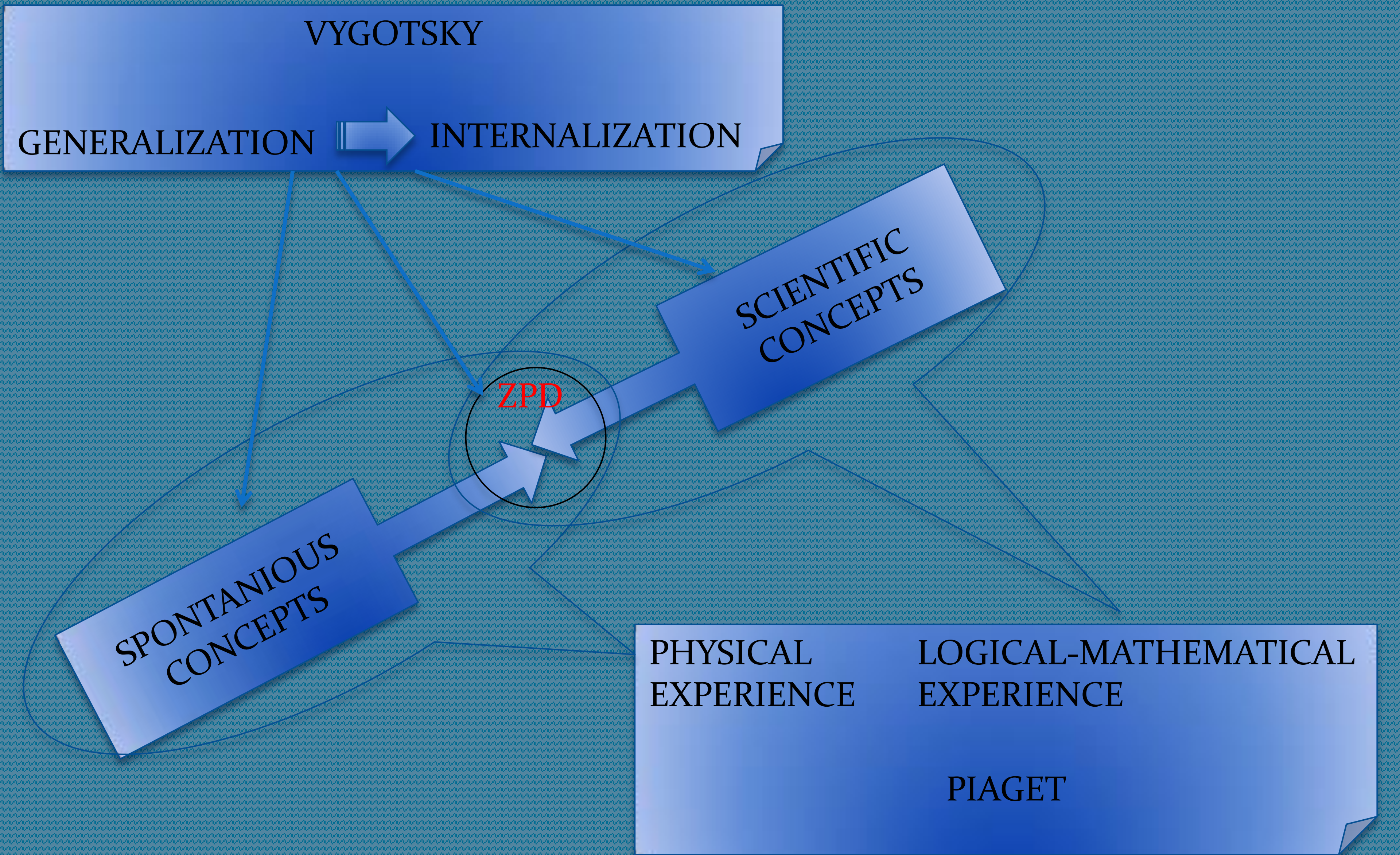
Chapter 6. Discussions

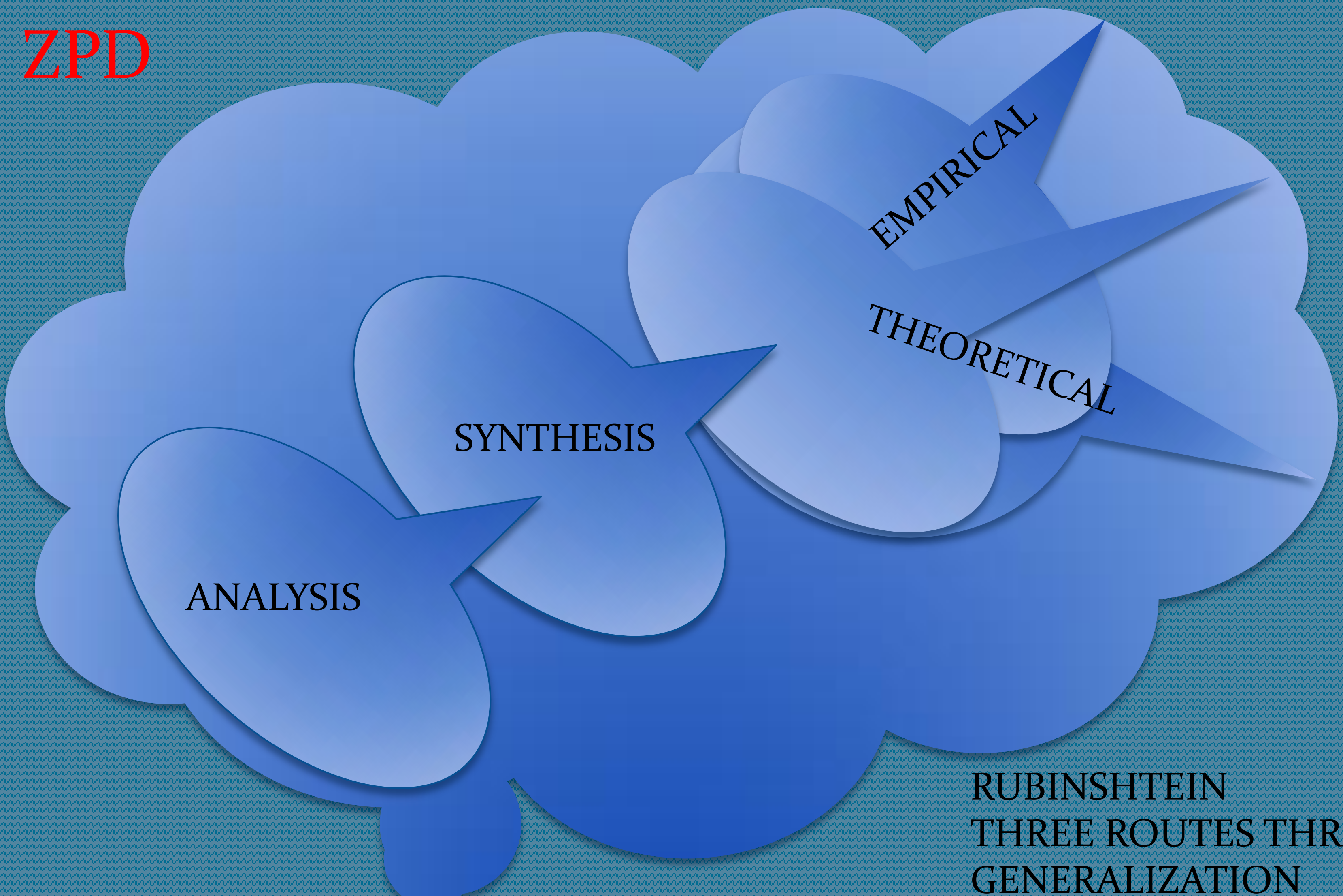
- What if the practice of generalization is interpreted from a different theoretical perspective?

My interpretation of the **CHAT METHAFOR**



What theories will be involved in this study?

ZPD



RUBINSHTEIN
THREE ROUTES THROUGH
GENERALIZATION

Leontiev: Activity Theory

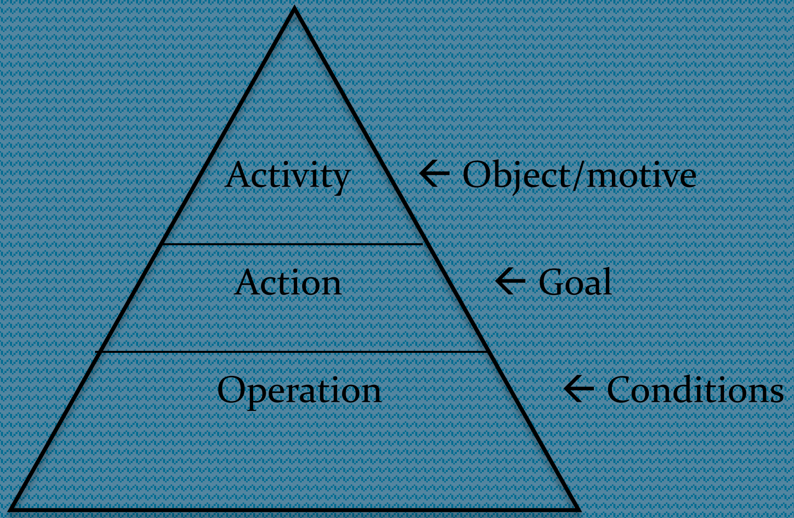
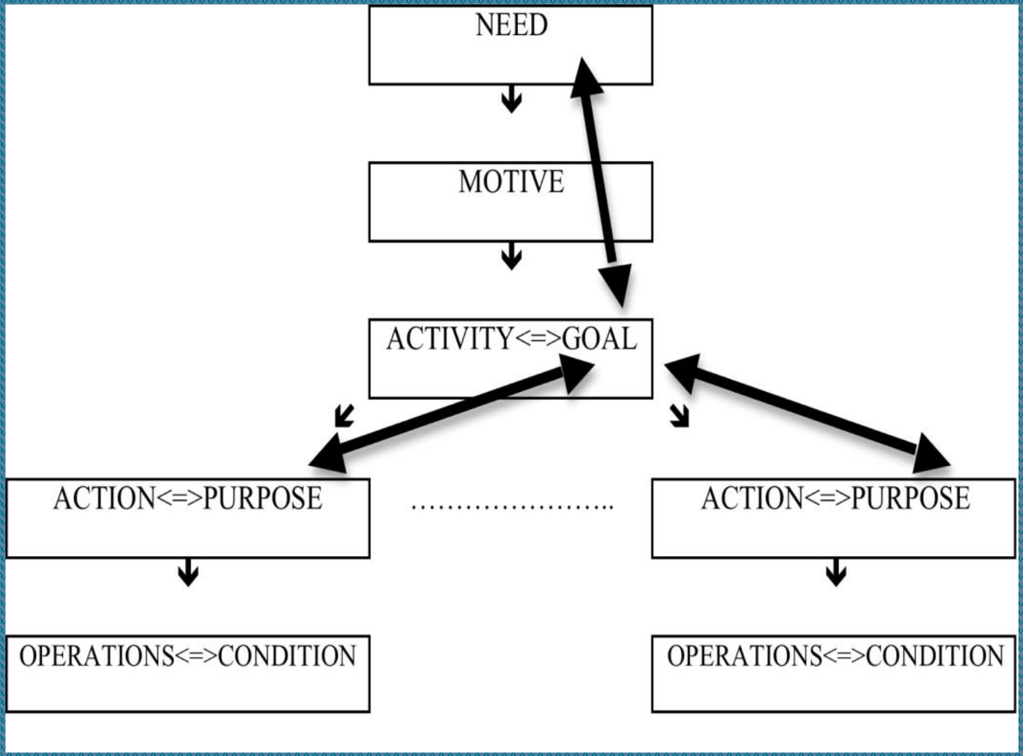


Figure 2.1 Activity System (Harry, 2008, p.119)



The practice of generalization in mathematics

Problem, question, surprise, contradiction, interest. Etc.

Solve a mathematical situation

GENERALIZATION<=> (a) to identify something general that is already known in particular cases and apply the general to the particular, and (b) to find something general that is not known from isolated and particular cases Krutetskii (1976)

ANALYSIS<=>Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose (Krathwohl, 2002, p. 215)	SYNTHESIS<=>Putting elements together to form a novel, coherent whole or make an original product (Krathwohl, 2002, p. 215)	ABSTRACTION<=> (a) Empirical abstraction: deriving knowledge from the properties of objects; (b) Pseudo-empirical abstraction: deriving knowledge from the properties of the objects that were introduced by acting on objects; (c) Reflective abstraction: drawing properties from mental or physical actions and project these actions at a higher level of thought (Dubinsky, 1991)
OPERATIONS	OPERATIONS	OPERATIONS

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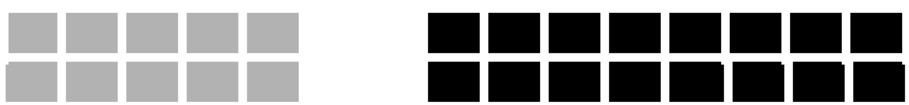
•Schifter, D. (2009). Representation-based proof in the elementary grades. In D. Stylianou, E. Knuth, & M. Blanton (Eds.), *Teaching and Learning Proof Across the Grades: A K-16 Perspective.* New York, NY: Routledge, pp. 71-86.

Paul: I know the sum is even because my older sister told me it always happens that way.
Zoe: I know it will add to an even number because $4+4=8$ and $8+8=16$.
Juan: Also, $6+12=18$ and $32+20=52$.
Eva: We really can't know! Because we might not know about an even number and if we add it with 2 it might equal an odd number!
James: We can never know for sure because the numbers don't stop.
Claudia: We don't know because numbers don't end. One million plus one hundred. You can always add another hundred.
Melody: Your answer will be even because you are using even numbers.

As Melody spoke, she, she pointed to arrangements of cubes in front her.



Then she continued:

Melody: This number is in pairs (pointing to the light colored cubes), and this number is in pairs (pointing to the dark colored cubes), and when you put them together, it's still in pairs.

.....

Melody: Because 2s don't get to odds. If they're two even numbers, they're both counted by 2s, and if you put them together, you keep counting by 2s, and that always equals an even number.
Eva: So she's saying she already knows it that it always equals even.

Reflective abstraction

Proposition: If m and n are even numbers, then their sum, $m + n$, is an even number.

Proof: If m and n are even, then, by definition, there are some integer k and some integer j such that

$$m = 2k \text{ and } n = 2j$$

Then

$$m + n = 2k + 2j$$

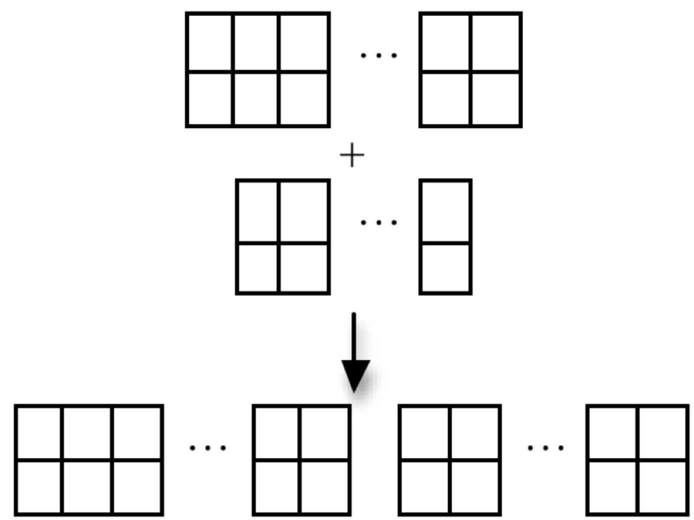
Applying the distributive law,

$$m + n = 2(k + j)$$

Thus, $m + n$ is equal to 2 times an integer, Establishing that the sum is an even number

Two even numbers added together give an even number.

Since they're two even numbers, they're both counted by 2s.



When you put them together, you keep counting by 2s.

Figure 4.1 Mathematician's proof versus a variation of Melody's proof

• Shai Caspi & Anna Sfard. Spontaneous meta-arithmetic as the first step toward school algebra. (2012)

"to find a certain place in the sequence I need the place that I found – it better be round – and then 3 – or any other number that is the regularity – times what must be added to the number you have now and then to add the product of the regularity and what you still need, and that's it"

(7th grade student)

Pseudo-empirical abstraction

Syntheses that lead to empirical and pseudo-empirical abstraction

MR: What about for the second part, is three divided by zero defined? Yah…

St_1: I did multiplication or, inverse multiplication so if you have as three or *x* equals three there is not value of *x* that you can put times zero to make it equal.

MR: Ok, there is nothing we can multiply by zero to get three. Does anyone disagree is there any alternate answers?…Yah..

St_2: I just said that you can also use the "how many groups" method to explain it, that three, there is no number you can divide three of then in zero groups, there is no number in that group, nothing can, that doesn't have like a value, so it's not defined.

MR: Okay, can you say that again? So, which interpretation…

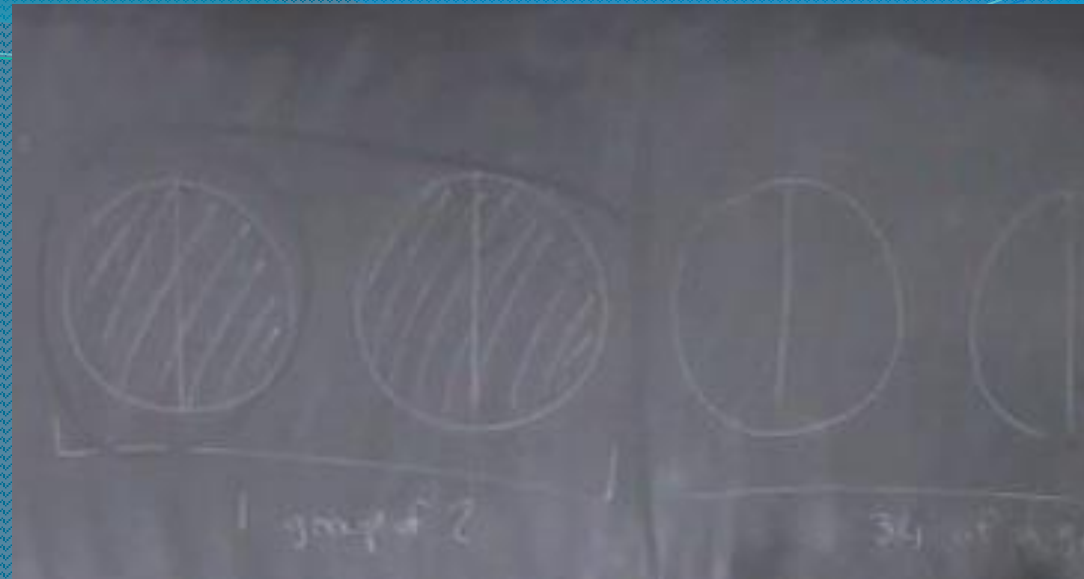
St_2: So, when you divide three up into zero groups, you can't divide three up into zero groups, there would be… the answer isn't zero, is not anything because you not dividing the number, you can't divide the number into zero groups. It is not defined because there is no any answer.

MR: Okay, there is no way to actually do that procedure. Yah…

St_3: And then if you use the "how many in each group", you can say… when three objects are divided into zero groups, is just the same thing like that, if you have zero groups the three is not going anywhere, so again it is undefined. So, I think you can use either interpretation as well as the multiplication to inverse.

Is this a representation of "how many groups" for

$$3\frac{1}{2},\ 2?$$



St: This took me so long to understand, which is why I really get it now.. But, this is the whole. And so, you have three and one half of your whole. And instead of dividing by two, you're taking how many groups of two are there in the whole. And so, you can see that this is a group of two and then what you have left is three fourths of one group of two. So how many groups of two you have left? There is one and three fourths groups of two. So, did it answer the question?

St: Also for me, that line that was center really thrown me off a little bit because I was like … with the imbedded, dividing them by two, is like how many parts … I would get rid of those.

MY PROBLEM

Does the above interpretation of the generalization practice represent a healthy understanding of the CHAT?

How does CHAT provide a new perspective for answering my research questions?